Preface

Th integer quantum Hall effect was discovered by von Klitzing on February 5th, 1980. A paper describing this result was submitted to *Physical Review Letters* in May, and published on August 11th, 1980. I first saw this paper at the Oji Seminar on semiconductor physics in strong magnetic fields held in Hakone, Japan beginning on September 9th, 1980. At a specially arranged night session in this seminar von Klitzing gave a talk on this phenomenon, and reprints of the *Physical Review Letters* paper were handed out for the participants. The participants were quite impressed by the fact that a measurement in solid-state physics gave such a precise measurement of the fine structure constant. However, few anticipated that this discovery would lead to the discovery of the fractional quantum Hall effect, which brought various new concepts into condensed-matter physics. The present book reviews the outcome of the studies on the quantum Hall effect which were launched by this great discovery.

The quantum Hall effect, which is the theme of this book, consists of the integer quantum Hall effect and the fractional quantum Hall effect. What von Klitzing discovered was the former effect. As a Japanese, I regret that this effect was not discovered in Japan. As a matter of fact, a theoretical calculation of the Hall resistivity by Ando, Matsumoto and Uemura at the University of Tokyo in 1975 had already suggested the quantization of the Hall resistivity. Wakabayashi and Kawaji of Gakushuin University measured the Hall resistivity in 1976; however, their lucky star did not shine on them. The effect was first considered as a way to measure the fine structure constant, but from the viewpoint of condensed-matter physics, it served to deepen our understanding of the connection between the electron wave function and the magnetic field.

The fractional quantum Hall effect, the younger brother of the integer quantum Hall effect, has proved to be more productive. In the course of the investigations, it was established that exact diagonalization of the Hamiltonian of a small finite-size system provided a quite effective method for the investigation of a macroscopic system, and a quite revolutionary theory was published, in which a very good many-body wave function is written down for this strongly correlated electron system. Following these pioneering works, many new concepts, or old concepts that had only existed in the brains of theoretical physicists, were shown to apply in this two-dimensional system, and they were actually observed. These concepts include anyons, i.e. particles with fractional statistics, fractionally charged quasiparticles, skyrmions, and Tomonaga–Luttinger liquids. The development of a new theoretical method, the transmutation of statistics by flux attachment, is also noteworthy. These accomplishments brought about by the study of the fractional quantum Hall effect have contributed tremendously to the progress in condensed-matter physics since 1980.

This book has been written to introduce this attractive discipline of condensed-matter physics to a wide range of researcher in physics in the hope that more people will participate in research on this subject. Of course, twenty years of intensive investigation by many researchers around the world appear to have clarified the essence of the phenomena. However, it happens frequently that suddenly something new comes out of a system that seems to have been fully studied.

This book is based on my lecture notes prepared for the graduate courses at Kyushu University and the University of Tokyo, but these notes have been heavily revised for publication. It was originally published in Japanese in 1998, and in the course of preparing the English version, Chap. 9 was added. The lectures were not intended for specialists, so a reader who has learned quantum mechanics up to second quantization should be able to understand this book. I expect that even a graduate student who is majoring in experimental physics will be able to understand it. This book is not a thorough review of the quantum Hall effect; rather, it is more like a textbook. The references are restricted to a minimal set. The subjects are also restricted: I have only included material that is necessary to a gain a correct overview of the quantum Hall effect. On the other hand, related materials that will help readers to understand the quantum Hall effect are also included, so that this book has some element of a textbook in condensed-matter physics. Most chapters have exercises. In many of these exercises the reader is asked to reproduce the equations in the text, the derivation of which is omitted in the main text. The solutions are given at the end of the book.

In this book the magnetic field is always in the positive z direction with strength B. The charge of an electron is written as e and is negative, i.e. $e < 0$. SI units are used, and I have tried to include the Planck constant $\hbar$ explicitly, although in the literature it is usually omitted. Books cannot be free from errors. For errors found after publication, errata will be published on the web site http://dbs.c.u-tokyo.ac.jp/~yoshioka/eqhebood.html.

Finally, I would like to thank all those who made publication of this book possible.

Tokyo, *Daijiro Yoshioka*
August 2001